Session #5
2D Mechanisms: Mobility, Kinematic Analysis & Synthesis

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Today’s Agenda

• Collect assignment #2
• Begin mechanisms
  – Mobility / degrees of freedom
  – Some basic mechanisms
  – Some basic synthesis and analysis techniques
  – MathCad preliminaries
  – Homogeneous Transformation Matrices
  – Solving systems of equations
  – Kinematic analysis
  – Mechanism synthesis
• Distribute & discuss assignment #3
• Distribute graded assignment #1
2D Mechanisms

- Where are they found?
- What do they do?

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http://workingmodel.design-simulation.com/
2D Mechanisms

- Where are they found?
- What do they do?
2D Mechanisms

• Where are they found?
• What do they do?
Two Goals in our Study of 2D Mechanisms

• Analysis
  – given a mechanism, predict its behavior
  – DOF, position, velocity, acceleration

• Synthesis
  – given a desired behavior, develop a mechanism
Degrees of Freedom

• DOF = the number of independent parameters required to completely define the entity’s positioning
• If a body is constrained to a plane, how many DOF does it possess?
  • If a pin joint is added?
  • If a sliding joint is added?
  • If it rolls without slipping?
Mobility Criteria

- Kutzbach criterion (to find the DOF)
  \[ F = 3(n-1) - 2j \]

- Grübler criterion (to have a single DOF)
  \[ 2j - 3n + 4 = 0 \]
Concept Question

• How many degrees of freedom does this mechanism possess?

1) 1 DOF
2) 2 DOF
3) 3 DOF
4) 0 DOF
Concept Question

• How many degrees of freedom does this mechanism possess?

1) 1 DOF
2) 2 DOF
3) 3 DOF
4) 0 DOF
Degrees of Freedom

• How many degrees of freedom does this mechanism possess?

We’ll cover 3D mechanisms later

Rear suspension of a Honda Accord

Figure removed for copyright reasons.
The 4 Bar Mechanism

• Use working Model to arrange a mechanism in which neither link can rotate a full turn

• Use working Model to arrange a mechanism in which rotating the input link a full turn causes the output link to:
  – Oscillate
  – Rotate a full turn
    • Same direction
    • Opposite direction
Parallelogram Linkage

• This object is connected to links that are parallel and of equal length
• What can we say (qualitatively) about the motion of the body (bce)?
3 Position Synthesis

- Say we want a mechanism to guide a body in a prescribed way
- Pick 3 positions
- Pick two attachment points
- The 4 bar mechanism can be constructed graphically
Concept Question

• If you do not specify the attachment point, how many positions can you specify and still generally retain the capability to synthesize a mechanism?

1) 3
2) 4
3) 5
4) > 5
Slider-Crank Mechanism

- Kinematically the same (in the limit) as a 4 bar mechanism with a very long output crank

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Inversion

- Take a mechanism
- Change which link is "ground"
- A new mechanism results
Velocity Analysis

• This object is translating and is NOT rotating
• The velocity if point “b” is indicated by a green arrow
• What do we know about the velocity of point “e”? 

Velocity Analysis

- This object is translating and ALSO rotating about “b” at rate $\omega$
- The velocity if point “b” is indicated by a green arrow
- Can we construct the velocity of vector of point “e”?
Velocity Analysis

- This object is a rigid body.
- The velocity of point “b” is indicated by a green arrow.
- The velocity of point “e” is indicated by a blue arrow.
- Can the vector “e” take any value, or are there some restrictions on values it might have?
Homogeneous Transformation Matrices

- How can we mathematically express the motions of a rigid body?
  - Translations
  - Rotations
  - Both
- Matrix multiplication will do the trick

\[
\text{body'} := \begin{pmatrix}
\cos(\theta) & -\sin(\theta) & 0 & \delta_x \\
\sin(\theta) & \cos(\theta) & 0 & \delta_y \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{pmatrix} \cdot \text{body}
\]
MathCad: Preliminaries

• Assigning a value to a scalar variable
• Assigning a value to a vector or matrix
• Defining a function
• Displaying results
  – Numerically
  – Graphically
MathCad: Useful Capabilities

- Taking Derivatives
- Integration
- Symbolic manipulation
- Solving non-linear equations
- Solving blocks of non-linear equations including constraints
- Minimizing / maximizing under constraints
- Making animations
- Processing images
HTM MathCad Demonstrator

- See HTM_DEMO_2D v2.xmcd
A Challenge

• Can you modify the HTM Demonstrator to simulate the operation of a 4 Bar mechanism?
  – Specify input and output crank geometry
  – Specify an input crank angle
  – Solve for an HTM that satisfies the physical constraints created by the cranks
Assignment #3

• Do some problems from the handout
• Analyze the kinematics of a robot’s leg mechanism
• Take some measurements of
  – Motion
  – Force
• Synthesize a mechanism (your choice)
Next Steps

• Start Assignment #3
• Lab tomorrow, Friday 24 FEB
• Next Class Tues 28 FEB