Session #6
2D Mechanisms: Synthesis

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Today’s Agenda

• Discuss assignment #2
• Mechanism synthesis
  – Solving systems of equations
  – Kinematic analysis
  – Position synthesis
  – Path generation
  – Employing constraints
Newton-Raphson Method

- Make a guess at the solution
- Make a linear approximation of a function by finite difference
- Solve the linear system
- Use that solution as a new guess
Secant Method

- Similar to Newton’s method
- But it uses the current and the last iterate to estimate the slope of the function
- Does not converge in as few iterations
- But requires fewer function evaluations
Bisection Methods

- Given an interval in which a solution is known to lie
- Look in the middle and determine which half has the root
- Iterate until the remaining interval is small enough
Solving Non-Linear Equations

- If there are many solutions, which solution you find will depend on the initial guess

\[ x_{\text{guess}} := 10 \quad x_{\text{root}} := \text{root}(y(x_{\text{guess}}), x_{\text{guess}}) \]

\[ x_{\text{guess}} := -3 \quad x_{\text{root}} := \text{root}(y(x_{\text{guess}}), x_{\text{guess}}) \]
• If you seek to find a root of a function \( f(x) \), and you use the Newton Raphson method.

• What are the possible outcomes?

• Choose all the numbers that apply:
  1) You find a solution no matter what initial guess you use
  2) You find a solution only after trying many different initial guesses
  3) You cannot find a solution because none exists
  4) You cannot find a solution even though one exists, even with many, many initial guesses
Newton-Raphson in Many Variables

\[ y(x_1, x_2) := 3 + x_2 + x_1 - \sin(x_2) \cdot (x_1 - 2)^2 + 0.2 \cdot (x_1 - 3)^3 \]

\[
\begin{align*}
x_1 &:= -3 \\
x_2 &:= 0
\end{align*}
\]

Given

\[ y(x_1, x_2) = 0 \]

\[ \text{soln} := \text{Find}(x_1, x_2) \]

\[ \text{soln} = \begin{pmatrix} 0.291 \\ -0.369 \end{pmatrix} \]

- Make a guess at the solution
- Make a linear approximation of a function
- Solve the linear system
- Use that solution as a new guess
• If you seek to find the solution to the system of equations.
• What are the possible outcomes? (depending on the values in the matrix $A$)
• Choose all the numbers that apply:

1) You find a solution no matter what initial guess you use
2) You cannot find a solution because none exists
3) You find many different solutions satisfy the equations
• If you seek to find the solution to the system of equations.

\[(x - c_{1x})^2 + (y - c_{1y})^2 = r_1^2\]

\[(x - c_{2x})^2 + (y - c_{2y})^2 = r_2^2\]

• What are the possible outcomes? (depending on the values \(c_{1x}, r_1\) etc.)

• Choose all the numbers that apply:

  • You cannot find a solution because none exists
  • After much searching, you find one solution and no others
  • After much searching, you find two solutions and no others
  • You find many, many solutions that satisfy the equations
Analysis of 2D Mechanisms

- If I assume the input crank rotates at constant angular velocity
- What does the rest of the mechanism do?
- Here are two ways to find out

(Mathcad demo)
A Challenge

• Can you modify the HTM Demonstrator to simulate the operation of a 4 Bar mechanism?
  – Specify input and output crank geometry
  – Specify an input crank angle
  – Solve for an HTM that satisfies the physical constraints created by the cranks
Concept Question

• If you do not specify the attachment point, how many positions can you specify and still generally retain the capability to synthesize a mechanism?

1) 3
2) 4
3) 5
4) >5
3 Position Synthesis

- Say we want a mechanism to guide a body in a prescribed way
- Pick 3 positions
- Pick two attachment points
- The 4 bar mechanism can be constructed graphically
Computer Aided 3 Position Synthesis

- Express the same basic ideas from the graphical construction in mathematical form
- Solve using a root finder

(Mathcad demo)
3 Position Synthesis: More Possibilities

- Allow the position where the link attaches to the moving body to vary
- How does that affect the required position of the fixed end of the link?
- Designing a 4 bar mechanism entails choosing two of these pairs of points
Concept Question

• How many points can you specify and still generally retain the capability to synthesize a mechanism?

1) 4
2) 5-7
3) 7-9
4) >9
4 Position Synthesis: Why Can’t I Find a Solution?

• The first 3 positions are all parallel
• This seems to call for a parallelogram linkage
• The last position is far from parallel
• This is hard to do
Computer Aided Path Generation

- Define a set of points through which a location on a moving body should travel
- Allow this point to be freely selected on the moving body
- Allow the body to rotate as needed
- Solve the system of equations
Next Steps

- Finish Assignment #3
- Next Class Thurs 2 MAR at MIT Museum
- Lab Friday 3 MAR