Try each of the questions; they will be given equal value. You may use theorems from class, or the book, provided you can recall them correctly!

**Problem 1**

Let \( f : [0, 1] \rightarrow \mathbb{R} \) be a continuous real-valued function. Show that there exists \( c \in (0, 1) \) such that
\[
\int_0^1 f(x) \, dx = f(c).
\]

**Problem 2**

(This is basically Rudin Problem 4.14)

Let \( f : [0, 1] \rightarrow [0, 1] \) be continuous.

1. State why the map \( g(x) = f(x) - x \), from \([0, 1]\) to \(\mathbb{R}\) is continuous.
2. Using this, or otherwise, show that \( L = \{ x \in [0, 1]; f(x) \leq x \} \) is closed and \( \{ x \in [0, 1]; f(x) < x \} \) is open.
3. Show that \( L \) is not empty.
4. Suppose that \( f(x) \neq x \) for all \( x \in [0, 1] \) and conclude that \( L \) is open in \([0, 1]\) and that \( L \neq [0, 1] \).
5. Conclude from this, or otherwise, that there must in fact be a point \( x \in [0, 1] \) such that \( f(x) = x \).

**Problem 3**

Consider the function
\[
f(x) = \frac{-x(x+1)(x-100)}{x^{44} + x^{34} + 1}
\]
for \( x \in [0, 100] \).

1. Explain why \( f \) has derivatives of all orders.
2. Compute \( f'(0) \).
3. Show that there exists \( \epsilon > 0 \) such that \( f(x) > 0 \) for \( 0 < x < \epsilon \).
4. Show that there must exist a point \( x \) with \( f'(x) = 0 \) and \( 0 < x < 100 \).

**Problem 4**

If \( f : \mathbb{R} \rightarrow \mathbb{R} \) and \( g : \mathbb{R} \rightarrow \mathbb{R} \) are two functions which are continuous at 0, show that the function
\[
h(x) = \max\{f(x), g(x)\}, \ x \in \mathbb{R}
\]
is also continuous at 0.