Course Outline: 18.085 Computational Science and Engineering

1. Special matrices K, T, B, C
   symmetric tridiagonal, invertible or singular
   pivots and free or fixed boundary conditions

2. Second differences from $1, -2, 1$
   $-u'' = f(x)$ becomes $Ku = f$
   $f =$ ones, $u =$ quadratic

3. Solving $Ku = f$
   $f =$ delta, $u =$ ramp
   inverses of K and T: discrete Green's function

4. $K = LDL^{\mathrm{T}}$ from elimination
   $K = Q\Lambda Q^{\mathrm{T}}$ from eigenvalues
   three-step solution of $u' = Ku$

5. Eigenfunctions $-y'' = \lambda y$
   eigenvectors $Ky = (2 - 2\cos\theta)y$
   sines, cosines, exponentials in y

6. Positive definite matrices: five tests
   $K = A^{\mathrm{T}}A$ and $K = A^{\mathrm{T}}CA$
   minimizing $P = \frac{1}{2}u^{\mathrm{T}}Ku - u^{\mathrm{T}}f$

7. Singular Value Decomposition $A = U\Sigma V^{\mathrm{T}}$
   norms of vectors and matrices
   numerical linear algebra: *lu*, *qr*, *svd*, *eig*

8. $A^{\mathrm{T}}CA$ for a line of springs
   displacements u from forces $f = A^{\mathrm{T}}CAu$
   elongation $e = Au$ and balance $A^{\mathrm{T}}w = f$

9. Oscillation from $Mu_{tt} + Ku = 0$
   solution by eigenvectors of $Kx = \lambda Mx$
   leapfrog and trapezoidal rules

10. Least squares gives $A^{\mathrm{T}}A\widehat{u} = A^{\mathrm{T}}b$
    solution by orthogonalization $A = QR$
    weights give $A^{\mathrm{T}}CA\widehat{u} = A^{\mathrm{T}}Cb$

11. Exam 1 on Lectures 1–9

12. Networks and incidence matrix A
    Kirchhoff's Current law $A^{\mathrm{T}}w = 0$
    graph Laplacian $A^{\mathrm{T}}A$ and $A^{\mathrm{T}}CA$

13. Trusses with 2N displacements
    mechanisms with $Au = 0$
    assembling A and K from each bar

14. Variances and covariances
    optimum weight $C = \Sigma^{-1}$
    recursive least squares (Kalman)

15. Continuous $A^{\mathrm{T}}CAu = -d/dx(c(x)du/dx) = f(x)$
    integration by parts for $(d/dx)^{\mathrm{T}}$
    weak form $\int cu'v'\,dx = \int fv\,dx$ for test functions $v(x)$

16. Galerkin's trial and test functions give $KU = F$
    linear finite elements $U_1\phi_1(x)$ to $U_n\phi_n(x)$
    assembly of K and F

17. Quadratic and cubic elements
    beam bending and 4th order problems
    B-spline for interpolation

18. Exam 2 on Lectures 10–16

19. Gradient and divergence
    potential u and stream function s
    equipotentials and streamlines

20. Laplace's equation div(grad u) = 0
    polynomial solutions from $x + iy$
    Cauchy-Riemann equations

21. Finite difference matrix K2D
    fast Poisson solver from sine transform
    odd-even reduction

22. Finite elements: linear in triangles
    assembly of $KU = F$ from element matrices
    boundary conditions and higher order elements

23. Fourier series: sines, cosines, e^{ikx}
    Gibbs phenomenon at jumps
    energy identity and decay rate $c_k = O(k^{-s})$

24. Series solution of the heat equation
    series solution of Laplace's equation on a circle
    delta function and analytic functions

25. Discrete Fourier Transform
    orthogonality of the Fourier matrix
    Fast Fourier Transform

26. Convolution and cyclic convolution
    Fast convolution by Fourier transform
    lowpass and highpass filters; equiripple filters

27. Fourier integrals and energy identity
    Green's function for input = delta function
    Heisenberg uncertainty principle and Gaussians

28. Deconvolution and integral equations
    circulant matrices and periodic filters
    autocorrelation and power spectral density

29. Exam 3 on Lectures 19–27

30. Wavelets and scaling functions
    multiresolution and perfect reconstruction
    compressed sensing using ℓ^1 and total variation norms

31. Analytic functions and Cauchy's Theorem
    Chebyshev points and fast transforms
    spectral methods of exponential accuracy